TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

STANDARDS VERSUS STANDARDS: THE EFFECTS OF DIFFERENT POLLUTION RESTRICTIONS ON THE FIRM'S DYNAMIC INVESTMENT POLICY

Peter M. Kort

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Peter M. Kort*

Economics Department

Tilburg University

P.O. Box 90153

5000 LE Tilburg

The Netherlands

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STANDARDS VERSUS STANDARDS: THE EFFECTS OF DIFFERENT POLLUTION RESTRICTIONS ON THE FIRM'S DYNAMIC INVESTMENT POLICY

In Helfand (1991) the effects of different pollution standards on the firm's resource allocation decisions are determined within a comparative static analysis. The purpose of this paper is to establish the implications of such standards within a dynamic framework. To do so a dynamic model of the firm is developed in which it is assumed that production causes pollution as an inevitable byproduct. Concerning its investment policy we suppose that the firm can choose between investing in productive capital goods and investing in abatement efforts.

It is shown that in some cases (future) abatement expenses have a negative impact on the present level of productive investment, even if the pollution standard is not binding at the moment. This implies a really dynamic optimal investment policy for the firm, which cannot be obtained within a comparative static analysis.

1. Introduction

The well known environmental debate between economists and policy makers nowadays is about what instrument should be used to reduce the level of pollution. The standpoint of economists is clearly stated by Ingham and Ulph (1991). They begin by argueing that, according to the analysis of Baumol and Oates (1971), efficiency requires that abatement methods must be exploited such, that marginal abatement costs are equal across all methods. This result leads to the argument that price-related regulations, such as taxes or marketable permits, are preferable to standards. The reason is that it would be an impossible task for the government to fix all standards such that marginal abatement costs are equal, while by imposing a tax marginal abatement costs are automatically equalized, because all polluters will abate such that marginal abatement costs equal the tax. On the other hand, practice shows that in most cases policy makers prefer standards rather than taxes or permits. In the literature papers can be found that try to find reasons for the occurrence of this paradox. For example, Buchanan and Tullock (1975) argue that firms will prefer emission standards to emission taxes because standards serve as a barrier to entry for new firms so that existing firms collect more profits. Their argument is based on the view that industry is able to exert its preference for a particular instrument because it is more likely to be well-organized than consumers. Ulph (1990) obtains that, by analyzing a multiple country game, standards should be preferred to taxes, because the use of standards permits greater commitment by producers and this allows them to earn higher surpluses. Moreover, in studying practical applications Hahn (1989) has shown that the performance of market based environmental systems is disappointing.

When economists refer to pollution standards, they almost universally mean uniform restrictions on pollution emissions. However, in practice, standards take many forms: not only emissions restrictions, but also restrictions on pollution per unit of output or per unit of an input, restrictions on the use of a polluting input, or mandated use of a particular pollution-control technology. In Helfand (1991) the implications of a range of standards are studied within a comparative static framework. The purpose of this paper is to extend Helfand's work by establishing the effects of standards in a dynamic environment.

In Section 2 the model is formulated, while in Section 3 we examine the effects of introducing five different kinds of standards, namely: a fixed level of emissions, a fixed level of emissions per unit of output, a fixed level of emissions per unit of an input, a fixed level of output, and a fixed level of an input. In Section 4 we compare the results of the different standards among each other and with the outcome of Helfand's analysis. Section 5 contains a summary and some general conclusions.

2. The Model

Consider a firm that owns a stock of capital goods K. In order to concentrate on pollution effects rather than capital-labor substitutions we assume that the labor-capital ratio is fixed:

$$L = \ell K \tag{1}$$

in which:

L : stock of labor

ℓ : labor-capital ratio ($\ell > 0$ and constant).

The firm produces a homogeneous output and production will be proportional to the inputs:[1)]

$$Q = qK = qL/\ell \tag{2}$$

in which:

Q : production rate

q : capital productivity ($q > 0$ and constant).

We assume that the sales level is an increasing function of production with decreasing marginal sales:

$$G(Q) = P(Q)Q \tag{3}$$

in which:

$P = P(Q)$: selling price per unit of production

$G = G(Q)$: sales rate, $G \geq 0$, $G' > 0$, $G'' < 0$, $G(0) = 0$.

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1) The results of this paper can also be obtained after assuming that Q is a concave function of K, except that then the stability analysis is more complicated.

G' being greater than zero says that the demand function is elastic with respect to the price, i.e. $-(P/Q)(dQ/dP) > 1$, and $G'' < 0$ is equivalent with assuming that $2P' < -P''Q$.[2)]
Due to the fixed labor-capital ratio, earnings, being equal to the difference between sales and labor costs, are a concave function of K. By using (1)-(3) this can be expressed as follows:

$$S(K) = (qP(qK) - w\ell)K \tag{4}$$

in which:

$S(K)$: earnings rate, $S \geq 0$, $S' > 0$, $S'' < 0$, $S(0) = 0$

w : wage rate ($w > 0$ and constant).

Capital stock decreases by depreciation and can be increased by productive investment:

$$\dot{K} = I - aK, \; K(0) = K_0 \tag{5}$$

in which:

I : rate of productive investment

a : depreciation rate.

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2) The major conclusions of this paper are not affected if, instead, we assume that the firm faces a horizontal output demand curve, i.e. P is constant.

The firm also produces pollution. Following Dasgupta (1982, pp. 152-154) (see also Van der Ploeg and Withagen (1991)) the emission-output ratio can be reduced by investment in clean technology. In this way the emission-output ratio, and thus the emission-capital ratio (cf. (2)), becomes a decreasing function of abatement investment:

$$E = \alpha(A)Q = e(A)K \tag{6}$$

in which:

E : amount of emissions

A : abatement investments

$\alpha(A)$: emission-output ratio, $\alpha' < 0$, $\alpha'' > 0$

$e(A)$: emission-capital ratio, $e(A) = q\alpha(A)$, $e' < 0$, $e'' > 0$.

According to Dasgupta (1982) abatement investments can arise if, for example, the firm uses different input mixes for production so that less pollution is generated. In practice we can think of switching to solar energy for the generation of domestic heating and thereby reducing SO_2 emission, or, in the case of crop production, substituting the use of pesticides by alternative techniques which involve, among other things, special planting combinations, the use of repellents and hormones, and the introduction of beneficial insects.

Note that $A = 0$ is associated with the production technology that would be chosen by a profit maximizing firm in absence of any environmental regulations. Hence, the emission-output ratio associated with this technology is $\alpha(0)$. The assumption of diminishing returns to abatement investments

($\alpha'' > 0$) seems to be realistic (see Kneese and Schultze (1975, Chapter 2) for practical examples).
Abatement investment are non-negative:

$$A \geq 0. \tag{7}$$

The firm is assumed to behave so as to maximize the net cash flow stream. After supposing that, due to adjustment costs, convex costs are associated to productive investments, and abatement investments face a horizontal supply curve, we arrive at the following objective function:

$$\text{maximize: } \int_0^{\infty} \exp(-rt)[S(K) - vA - C(I)]dt \tag{8}$$

in which:

- $C(I)$: costs of productive investment, $C(0) = C'(0) = 0$, $C'' > 0$
- r : discount rate
- v : price of a unit of abatement investment ($v > 0$ and constant).

To facilitate the analysis later on we introduce the following additional assumption concerning the shape of the emission-capital ratio function:

$$2(e')^2 < ee''. \tag{9}$$

3. The Constraints

Here, we study the implications of five kinds of different pollution standards. We start by incorporating a maximal emission standard and proceed by introducing restrictions on emissions level per unit of output, emissions level per unit of an input, output level, and input level, respectively.

But, in order to serve as a benchmark, we first derive the solution in case there are no environmental regulations. Then there is no incentive for abatement investments, implying that $A = 0$. Hence, the control problem left to be solved is given by (5) and (8) with $A = 0$. The current value Hamiltonian equals:

$$H = S(K) - C(I) + \lambda(I-aK). \tag{10}$$

The necessary conditions are:

$$\lambda = C'(I) \tag{11}$$

$$\dot{\lambda} = (r+a)\lambda - S'(K). \tag{12}$$

These conditions are also sufficient for optimality provided that the following transversality condition holds for every feasible solution $\tilde{K}$ (cf. Feichtinger and Hartl (1986)):

$$\lim_{t\to\infty} \exp(-rt)\lambda(t)[\tilde{K}(t) - K(t)] \geq 0. \tag{13}$$

From (11) and (12) we can derive:

$$\dot{I} = \frac{1}{C''(I)} \{(r+a)C'(I) - S'(K)\}. \tag{14}$$

The steady state follows from (5) and (14) and can be expressed as:

$$\hat{I} = a\hat{K} \tag{15}$$

$$S'(\hat{K}) = (r+a)C'(\hat{I}). \tag{16}$$

The determinant of the Jacobian of the system (5), and (14), which is evaluated in $(\hat{I},\hat{K})$, is negative so that the dynamics corresponds to a saddle point.

After solving the differential equation (12), substituting (11) into this relation, and using (16) as a fixed point, we obtain:

$$\int_t^{\infty} S'(K(s))\exp(-(a+r)(s-t))ds - C'(I(t)) = 0. \tag{17}$$

The first term of (17) stands for the discounted marginal earnings stream caused by an additional unit of investment at time "t". Here it is reckoned with the fact that capital depreciates and therefore increases only with $\exp(-a(s-t))$ at each time $s > t$. The second term represents the initial outlay that is required to raise the capital stock by one unit at time "t". Hence the left-hand side of (17) is equal to the benefit of an investment of one dollar and, according to business economics terminology, we denote this benefit by net present value of marginal investment (NPVMI) (see also Kort (1989, 1990)). From (17) we infer that the NPVMI equals zero. Therefore for the firm's investment policy to be optimal it is required that marginal earnings equal marginal expenses.

3.1. Standard as a Set Level of Emissions

Let Z_E be the numerical standard set when emissions are regulated by the amount of total pollution permissible per unit of time. From (6) we derive that the following constraint has to be imposed:

$$e(A)K \leq Z_E. \tag{18}$$

Now we need to solve the control problem represented by (5), (7), (8) and (18). We first note that A only occurs in the objective and in the constraints, but not in the system dynamics (5). Therefore, the problem can be solved by application of a two-step procedure (see e.g. Feichtinger and Hartl (1986, pp. 397-402) and Hartl (1988)).

Step 1:

Solve for every fixed K the static optimization problem:

$$\max_A \{-vA \,|\, e(A)K \leq Z_E;\ A \geq 0\}. \tag{19}$$

After deriving the Kuhn Tucker conditions we obtain that the solution of (19) is given by $A = A_E(K)$, where $A_E(K)$ equals:

$$A_E(K) = \begin{Bmatrix} 0 \\ A_{E2}(K) \end{Bmatrix} \text{ for } K \begin{bmatrix} \le Z_E/e(0) \\ > Z_E/e(0) \end{bmatrix}, \tag{20}$$

where $A_{E2}(K)$ is an implicit function that satisfies:

$$e(A_{E2}(K))K = Z_E. \tag{21}$$

The derivatives of this abatement function are given by:

$$A'_{E2}(K) = -e/e'K > 0 \tag{22}$$

$$A''_{E2}(K) = e\{2(e')^2 - ee''\}/\{(e')^3K^2\} > 0. \tag{23}$$

The "greater than"-sign in (23) is due to assumption (9). Hence A_E is a convex non-decreasing function of K.

Step 2:

Solve for $A = A_E(K)$ the control problem represented by (5) and (8). The Hamiltonian is given by:

$$H_E = S(K) - vA_E(K) - C(I) + \lambda_E(I-aK). \tag{24}$$

The necessary conditions are:

$$\lambda_E = C'(I) \tag{25}$$

$$\dot{\lambda}_E = (r+a)\lambda_E - S'(K) + vA'_E(K). \tag{26}$$

Because $A_E(K)$ is convex, these conditions are also sufficient for optimality provided that a transversality condition like (13) is satisfied.
Due to (25) and (26) we get:

$$\dot{I} = \frac{1}{C''(I)} \{(r+a)C'(I) - S'(K) + vA_E'(K)\}. \tag{27}$$

Now we carry out a (state-control) phase plane analysis for the differential equation system ((5), (27)). Due to (20), (22) and (27) we derive that the $\dot{I} = 0$ isocline jumps downwards when K equals $Z_E/e(0)$. From Figure 1 we obtain that there are three possible configurations.

[Place Figure 1 about here]

Because of (20) the $\dot{I} = 0$ isocline, which in Figure 1 is denoted by $\dot{I}_E = 0$, consist of two parts. For K less than $Z_E/e(0)$ it holds that $A_E(K) = 0$ (cf. (20)) and therefore the $\dot{I}_E = 0$ isocline coincides with the $\dot{I} = 0$ isocline of the unregulated case (cf. (14), (27)). Hence, if the equilibrium point is such that $\hat{K} < Z_E/e(0)$, then we have the same equilibrium point for the regulated and the unregulated case (see configuration (iii) in Figure 1). For $K > Z_E/e(0)$ the $\dot{I}_E = 0$ isocline of the regulated case is situated below the $\dot{I} = 0$ isocline of the unregulated case. Therefore, the equilibrium points differ (configurations (i), (ii)) and in

Figure 1 the equilibrium point of the unregulated case is denoted by ∘ and the equilibrium point of the regulated case by • .

In Figure 2 the three configurations are drawn in separate phasediagrams. Also here the saddlepoint of the unregulated case $(\hat{K},\hat{I})$ is denoted by ∘ and of the regulated case $(\hat{K}_E,\hat{I}_E)$ by • .

[Place Figure 2 about here]

In configuration (i) the equilibrium level of capital stock in the regulated case satisfies:

$$S'(\hat{K}_E) = (r+a)C'(a\hat{K}_E) + vA_E'(\hat{K}_E). \tag{28}$$

(28) says that in equilibrium marginal earnings equal the sum of marginal investment costs and marginal abatement expenses necessary to keep pollution equal to the standard level when capital stock increases marginally. It is easy to check that the determinant of the Jacobian is negative so that this equilibrium point is a saddlepoint as well. Compared to the equilibrium level in the unregulated case (cf. (15), (16)), here the capital stock is lower which is due to the abatement expenses that are forced by the standard.

In configuration (ii) the equilibrium level of capital stock equals $Z_E/e(0)$ and it satisfies:

$$(r+a)C'(a\hat{K}_E) < S'(\hat{K}_E) < (r+a)C'(a\hat{K}_E) + vA_E'(\hat{K}_E). \tag{29}$$

From the first inequality of (29) we infer that marginal earnings exceed marginal investment costs, so it would be optimal for the firm to grow further when no abatement investments are necessary. But when the firm grows beyond $Z_E/e(0)$, then abatement expenditures are needed to meet the standard (cf. (20)). Hence, marginal costs increase with the abatement costs and from the second inequality of (29) we obtain that this implies that total marginal costs exceed marginal earnings. This means that it is optimal for the firm to keep the level of capital stock equal to $Z_E/e(0)$. From the first inequality of (29) we also derive that the equilibrium level of capital stock in the unregulated case exceeds the one in the regulated case.

In configuration (iii) the equilibrium point is given by:

$$S'(\hat{K}_E) = (r+a)C'(a\hat{K}_E). \tag{30}$$

Here the equilibrium level is such ($\hat{K}_E < Z_E/e(0)$) that no abatement expenditures are necessary to meet the standard. Therefore no abatement costs are contained in the marginal earnings-cost relation of (30). Comparing (16) and (30) shows that the equilibrium levels of the regulated and the unregulated case are the same.

We see that imposing the emission standard only reduces the equilibrium level when $\hat{K}$ is higher than $Z_E/e(0)$. This brings us to the conclusion that especially large firms are influenced by an emission standard. This feature was also found in Helfand (1991).

From Figure 2 we obtain that in the configurations (i) and (ii) the investment level is always lower in the regulated case. In configuration (iii) this only holds for large values of capital stock, namely for those values where abatement investments are needed to satisfy the standard. In configureations (i) and (ii) it turns out that investments are lower even when abatement expenditures are not yet required. This behavior can be confirmed by a net present value rule. Suppose that the firm starts out with a capital stock lower than $Z_E/e(0)$ and that this level is reached at time "t_E". Then, from the optimality conditions we can derive the following expression for the net present value of marginal investment (NPVMI):

$$\int_t^{\infty} S'(K(s))\exp(-(a+r)(s-t))ds - \int_{t_E}^{\infty} vA_E'(K(s))\exp(-(a+r)(s-t))ds - C'(I) = 0. \tag{31}$$

Because the second term is positive the investment level must be reduced (compared to the unregulated case (cf. (17))) to keep the NPVMI equal to zero. This equation confirms that at each moment of time the firm reckons with future abatement expenditures when it determines its investment rate. The reason for this is that when the firm invests one dollar at time "t", capital stock increases with $\exp(-a(s-t))$ at each time $s > t$ implying that at each time $s > t_E$ additional abatement expenditures are necessary to keep pollution equal to Z_E.

When we consider a firm starting out with a capital stock below $Z_E/e(0)$ in configuration (iii), then K will remain below $Z_E/e(0)$. This implies that no future abatement expenditures are needed, so the second term in (31) disappears which means that the NPVMI-expression becomes equal to the one in the unregulated case (cf. (17)). Therefore the investment levels of the regulated and unregulated case coincide here.

3.2. Standard as Emissions per Unit of Output

Let Z_{EQ} be the standard expressed as a set level of pollution per unit of output. According to (6) this amounts to:

$$e(A)K \leq Z_{EQ}Q. \tag{32}$$

From (2) we obtain that this constraint can be rewritten into:

$$e(A) \leq qZ_{EQ}. \tag{33}$$

Abatement expenditures are costly and, therefore, it is optimal for the firm to put them as low as possible. In this way optimal abatement investments are given by:

$$A_{EQ} = \begin{Bmatrix} 0 \\ A_{EQ2} \end{Bmatrix} \text{ if } e(0) \begin{Bmatrix} \leq qZ_{EQ} \\ > qZ_{EQ} \end{Bmatrix}, \tag{34}$$

where A_{EQ2} satisfies:

$$e(A_{EQ2}) = qZ_{EQ}. \tag{35}$$

Hence, abatement expenditures are constant over time and also independent of the stock of capital goods. Abatement expenditures are not needed when the emission-output ratio without abatement efforts $(e(0)/q)$ already satisfies the standard. But the standard can also be so restrictive that over the whole planning period the firm must assign a constant amount of money to abatement investments in order to reduce the emission-output ratio. As said before A does not depend on K and thus the remaining control problem can be solved independently from A. Consequently, this leads to the same productive investment policy as in the unrestricted case and thus (15)-(17) also apply here.

From an economic point of view, the level of productive investment being unaffected by the emissions per unit of output standard can be explained by noting that here having an increased stock of capital goods does not have consequences for (future) abatement expenses. Therefore abatement costs do not occur in the NPVMI-relation (cf. (17)). Because abatement expenses are the same for a large and a small firm, an emission per unit of output standard seems to be unfavorable for small firms.

3.3. Standard as Emissions per Unit of a Specified Input

Two cases are possible here: regulating pollution per unit of capital goods or regulating pollution per unit of abatement investments. The first possibility leads to the following mathematical representation:

$$e(A)K \le Z_{EK}K. \tag{36}$$

After dividing both sides by K we conclude that imposing this constraint is similar to imposing the emissions per unit of output standard and, therefore, the results stated in the previous subsection also apply here.

Regulating pollution per unit of abatement investments leads to the following constraint:

$$e(A)K \le Z_{EA}A. \tag{37}$$

Since abatement investments are costly there is no reason for the firm to invest in abatement activities more than required by the standard. Hence, throughout the whole planning period A will be an implicit function of K, $A = A_{EA}(K)$, that satisfies:

$$e(A_{EA}(K))K = Z_{EA}A_{EA}(K). \tag{38}$$

From (38) we derive:

$$A'_{EA}(K) = e/(Z_{EA}-e'K) > 0 \tag{39}$$

$$A''_{EA}(K) = \{2e'Z_{EA} + K(-2(e')^2 + ee")\}e/(Z_{EA}-e'K)^3. \tag{40}$$

The sign of the second derivative is ambiguous because in the numerator the first term is negative, while the second term is positive due to assumption (9).

As step 2 we now need to solve the control problem represented by (5) and (8) for $A = A_{EA}(K)$. The Hamiltonian is given by:

$$H_{EA} = S(K) - vA_{EA}(K) - C(I) + \lambda_{EA}(I-aK). \tag{41}$$

Then the necessary conditions are:

$$\lambda_{EA} = C'(I) \tag{42}$$

$$\dot{\lambda}_{EA} = (r+a)\lambda_{EA} - S'(K) + vA'_{EA}(K). \tag{43}$$

From these two conditions we derive:

$$\dot{I} = \frac{1}{C''(I)} \{(r+a)C'(I) - S'(K) + vA'_{EA}(K)\}. \tag{44}$$

The steady state value of capital stock can be obtained from (5) and (44):

$$S'(\hat{K}_{EA}) = (r+a)C'(a\hat{K}_{EA}) + vA'_{EA}(\hat{K}_{EA}). \tag{45}$$

Comparing (15)-(16) and (45) leads to the conclusion that this steady state value is lower than in the unregulated case. The determinant of the Jacobian evaluated in the steady state equals $-a(r+a)C'' + S'' - vA''$. Notice that negativity of this determinant is not assured because one of the two terms in the numerator of A'' is negative. However, it is likely that negativity of this term is compensated by the other term in the numerator of A'' and by the first two terms of the determinant. Hence, also here we suppose that the dynamics corresponds to a saddlepoint.

From the optimality conditions we obtain the following NPVMI-relation:

$$\int_t^{\infty} \{S'(K(s)) - vA'_{EA}(K(s))\}\exp(-(r+a)(s-t))ds - C'(I(t)) = 0. \tag{46}$$

Due to the fact that the firm abates such that the standard is just satisfied (cf. (38)) an additional investment expenditure immediately requires extra abatement expenses. Therefore these are subtracted from marginal earnings in the NPVMI-relation, which implies that the level of productive investment is lower than in the unregulated case.

The difference with the emissions standard is that here throughout the whole planning period abatement expenses are required to meet the emissions per unit of abatement standard, while with the emissions standard this is only the case when capital goods exceed a certain level. Hence, unlike the emissions standard, the emissions per unit of abatement standard does not favor the small firms.
According to the analysis of Helfand (1991) the performance of three standards (emissions per unit of output and emissions per unit of either input) cannot be readily distinguished from each other. Here it holds that imposing standards that restrict emissions per unit of output and emissions per unit of capital goods lead to the same results. But the outcome of imposing a standard that restricts the emissions per unit of abatement input is quite different, because the latter results in abatement expenses that increase with capital goods (cf. (39)) while abatement expenses are constant in the cases of an emission per unit of output standard and an emissions per unit of capital goods standard.
We found out that the emissions standard favors the small firm, while the emissions per unit of output or per unit of capital goods standard favor the large firm. It seems that the emissions per unit of abatement standard does not discriminate any firm.

3.4. Standard as a Set Level of Total Output

The constraint to be added is now:

$$qK \leq Z_Q. \tag{47}$$

Because pollution, or pollution per unit output or input, is not directly restricted here, abatement expenses are not of any use to the firm, so throughout the whole planning period it holds that A = 0. Thus the control problem to be solved is represented by (5), (8) (with A = 0) and (47), where the latter is a pure state constraint. Of course it has to be imposed that $K(0) \le Z_Q/q$. By using the direct method (see Feichtinger and Hartl (1986)) the Lagrangian equals:

$$L = S(K) - C(I) + \lambda_Q(I-aK) + \mu_Q(Z_Q/q-K). \tag{48}$$

The necessary conditions are:

$$\lambda_Q = C'(I) \tag{49}$$

$$\dot{\lambda}_Q = (r+a)\lambda - S'(K) + \mu_Q \tag{50}$$

$$\mu_Q(Z_Q/q-K) = 0, \ \mu_Q \ge 0. \tag{51}$$

Due to the fact that the Hamiltonian is strictly concave in I it is regular. Now, from Corollaries 6.2, 6.3a of Feichtinger and Hartl (1986) and due to satisfaction of constraint qualification (6.17) of Feichtinger and Hartl (1986) we get that I and λ_Q are continuous.

To obtain the optimal investment policy we follow the approach of Feichtinger and Hartl (1986, pp. 218-219). First, notice that there are two possibilities:

a. $K(t) < Z_Q/q \ \forall t$.

This implies that $\mu_Q = 0$ so that the optimality conditions are the same as in the unregulated case. Consequently, the optimal investment policy is here also given by (17).

b. K reaches its upperbound at a certain point of time.

Starting at $K = K_0$ we have to find a trajectory that satisfies the constraint as well as the necessary conditions everywhere. To do so we intersect $K = Z_Q/q$ with the $\dot{K} = 0$ isocline and study the trajectory that ends in this intersection point, which is denoted by (K_Q, I_Q) (see Figure 3).

[Place Figure 3 about here]

If we choose for $K = K_0$ the corresponding investment rate on this trajectory then the point (K_Q, I_Q) is reached at a finite point of time t_Q. Then it makes sense to choose the control $I = I_Q$ for $t \in (t_Q, \infty)$.
The necessary conditions are of course satisfied for $t \leq t_Q$. For $t > t_Q$ it holds that $\lambda_Q = C'(I_Q)$. Hence $\dot{\lambda}_Q = 0$, and we obtain (after noticing that $K_Q < \hat{K}$ and $\hat{K}$ satisfies (cf. (16)): $S'(\hat{K}) = (r+a)C'(\hat{I})$):

$$\mu_Q = S'(K_Q) - (r+a)C'(I_Q) > 0. \tag{52}$$

Hence, also for $t > t_Q$ the necessary conditions are fulfilled. Because K and λ are always finite on this trajectory the sufficient conditions from

Theorem 7.5 in Feichtinger and Hartl (1986) are satisfied, which means that the constructed solution is really optimal.

From the figure we infer that, compared to the unrestricted problem, the investment rate is lower. Hence, like in the problem with the emissions standard, in determining its investment policy the firm seems to reckon with the fact that the maximal output level will be reached after a while. This is confirmed by the NPVMI-relation which has the following form:

$$\int_t^\infty S'(K(s))\exp(-(a+r)(s-t))ds - \int_{t_Q}^\infty \mu_Q(s)\exp(-(a+r)(s-t))ds - C'(I) = 0. \tag{53}$$

So, like in the previous solutions, the firms investment policy has a really dynamic structure. Therefore, such a result can not be obtained within a comparative static context (cf. Helfand (1991)). One of the results that coincide with Helfand is that the use of both inputs (K as well as A) decreases in this case.

3.5. Standard as a Set Amount of a Specified Input

This standard takes two forms. A maximum can be set on the stock of capital goods; alternatively, imposing a minimum level on the use of an abatement input captures the effect of imposing a particular pollution-control technology on a firm. The first possibility leads to the following mathematical representation:

$$K \le Z_K. \tag{54}$$

After comparing (47) and (54) we conclude that setting a maximum on the capital stock will have the same effect as restricting the level of total output and, therefore, the results stated in the previous subsection also apply here.

Imposing a minimum level on the level of abatement investment gives the following constraint:

$$A \geq Z_A. \tag{55}$$

The optimal control problem to be solved consists of the expressions (5), (8) and (55). Since abatement investments have to be paid for, the optimal policy is to put them as low as possible, i.e. $A(t) = Z_A$ for all t. Furthermore, the firm will apply the same productive investment policy as in the unregulated case, because abatement costs will not be influenced by an increase of capital goods. Hence, the level of productive investment satisfies the NPVMI-relation (17).

4. Comparisons of the Different Standards

Like in Helfand (1991) comparisons among the standards can only be made when they are normalized. Here we normalize the standards such that in each steady state the firm produces the same amount of pollution. Also, we consider only those solutions where the standard has the biggest impact. For example, in case of the emissions standard (E) we arrived at three solutions and here we pick that solution where at the end abatement expenses are needed to satisfy the standard.

As was noted in the previous section restricting the emissions per unit of output (EQ) and restricting the emissions per unit of capital goods (EK) lead to the same outcome and this also holds for the maximal output standard (Q) and the maximal capital stock standard (K). Further we derived that abatement expenditures are constant when the emissions per unit of output standard and the minimal abatement standard (A) are imposed. Because the productive investment policy coincides under these two standards and normalization requires that the amount of pollution must be the same in the steady state, the abatement expenditures must be the same too and thus the performance of standards EQ and A will be equal after normalization. This brings us to the conclusion that, after normalizing the standards, we have to consider four different solutions, namely the solutions resulting from imposing the emissions standard, the emissions per unit of output standard, the emissions per unit of abatement investment standard and the maximal output standard, respectively. Together with the unregulated case they are depicted in Figure 4, in which it is assumed that the firm starts out with a rather low level of capital stock.

[Place Figure 4 about here]

Because the productive investment policy is the same, the development of capital stock over time in the unrestricted case coincides with the solution where the emissions per unit of output are restricted. We see that over the whole planning period this emission per unit of output standard gives the highest level of capital stock. Because the firm does not spend money on abatement efforts in the case of a maximal output standard and the amount of emissions must be equal in the end, the level of capital stock will be mostly reduced in this case.[3)]

As just mentioned, restricting emissions per unit of output gives the highest level of capital stock. Then abatement expenditures must also be at the highest level, because emissions are the same in the end. In the case of an emissions standard we first have a period of zero abatement investment and it becomes positive as soon as the standard level of emissions is reached. Contrary to this, in the case of the emissions per unit of abatement investment standard the firm will carry out abatement investments during the whole planning period.

Of course, the amount of emissions reaches the highest level in the unrestricted case. For the different standards it holds that the amount of emissions is the same in the end, due to the normalization. Unlike the other standards, this emissions level is already reached within a finite time period when an emissions standard is imposed. At the start of the planning period emissions are mostly reduced in the case of an emissions per unit of output standard, because imposing the latter results in a high constant level of abatement expenditures over the whole planning period.

In case that, like in our model, the abatement input does not contribute to production, the analysis of Helfand (1991) leads to the conclusion that standards can be divided into two groups which give the same performance. The first group, consisting of the standards E, EQ, EK, EA and A, leads to

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3) The relative levels of capital stock between the emissions standard and the emissions per unit of abatement investment standard cannot be determined. On the diagram it is assumed that these levels are equal in the end and that in the beginning capital stock grows faster in case of an emissions standard. The latter seems to make sense because then there are no abatement expenditures in this case.

higher levels of input, output and profits than the second group, which contains the standards Q and K. From our dynamic analysis it can be concluded that, contrary to Helfand (1991), standards E and EA lead to lower levels of input and output than the standards EQ, EK and A. But the profit level of standard E is higher than that of all other standards. This is because the solutions of these standards all satisfy the emissions constraint so that they are feasible in case of an emissions standard. But, apparently they are not optimal, which implies that their profit levels are lower than that of the optimal solution in case of an emissions standard.

5. Summary and Conclusions

In this paper we studied the dynamic behavior of the firm under different pollution standards. In the economic theory a pollution standard is synonym to restricting the amount of emissions, but in practice standards can take many forms. Here we follow the approach of Helfand (1991) who considered standards that restrict emissions, emissions per unit of output, emissions per unit of an input, output and use of an input, respectively. We extend Helfand's comparative static analysis by considering a dynamic framework.

We developed a dynamic model of the firm in which output was produced by labor and capital stock in fixed proportion. Where labor could be adjusted freely, capital stock could be raised by (productive) investments which are subject to adjustment costs. The production process also generates pollution as an inevitable byproduct. If, in order to satisfy a pollution standard, the amount of emissions must be reduced, we assumed that this could not only be done by decreasing the production rate but also by carrying out abatement investments through which the emissions-output ratio could be diminished.

It turned out that the firm's productive investment policy had a really dynamic structure. In all cases the level of productive investment could be determined by a dynamic investment decision rule that is based on the concept net present value of marginal investment. From our analysis we infer that the firm's productive investment policy is not changed when an emissions per unit of output standard, or an emissions per unit of capital goods standard, or a minimal abatement investment standard is imposed. In case of an emissions standard, a maximal output standard and a maximal capital goods standard, already at the beginning of the planning period productive investment is reduced in order to anticipate on the time period that the standard becomes binding.

Abatement expenditures turn out to be zero when a maximal output standard or a maximal capital goods standard is imposed, because then there is no incentive for the firm to reduce pollution. In case of an emissions per unit output standard, an emissions per unit of capital goods standard and a minimal abatement investment standard, abatement investments are constant over time. When emissions are restricted abatement investments are zero as long as the amount of emissions is below standard level, while abatement investments are always positive in case of an emissions per unit of abatement investment standard.

The advantage of this work compared to Helfand (1991) is that our analysis is dynamic. The application of optimal control to this general problem yields insights regarding the possible intertemporal effects of pollution standards. This is important since policy makers need to be concerned about "short-run" effects of policy that occur before a final equilibrium is reached. Since the time required to reach dynamic equilibrium varies and in some instances could be significant, characterization of the possible approach paths paints a picture of what life might be like in a time frame that may not be that "short". This kind of knowledge may be useful as input to policy decisions.

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Figure captions

Figure 1. Three possible configurations in the state-control phasediagram.

Figure 2. The optimal solution in the cases (i)-(iii): unregulated case (•••), regulated case (——).

Figure 3. The optimal solution of the problem with the maximal output standard (——) and of the problem with no constraints (•••).

Figure 4. Capital stock, abatement investments and amount of emissions as functions of time in five different solutions: unregulated solution (•••), emissions standard solution (---), emissions per output standard solution (-•-•-•), emissions per abatement investment standard solution (vvv), maximal output standard solution (>>>).

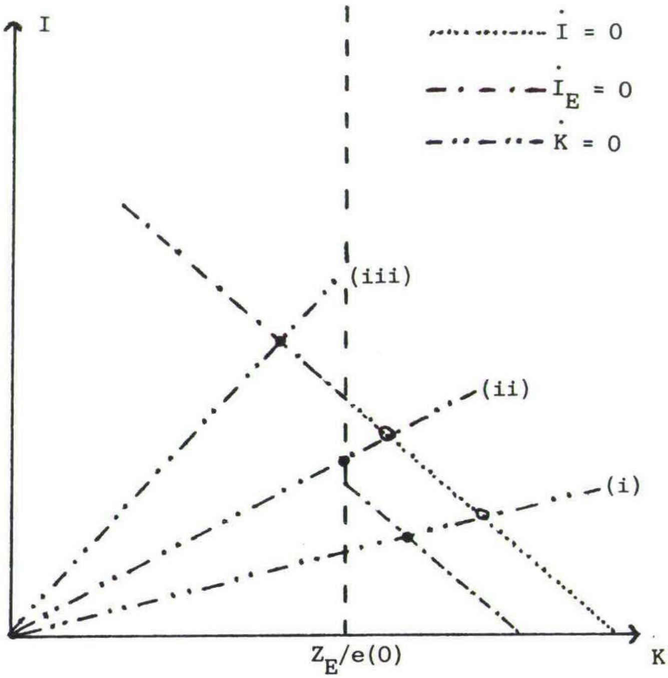
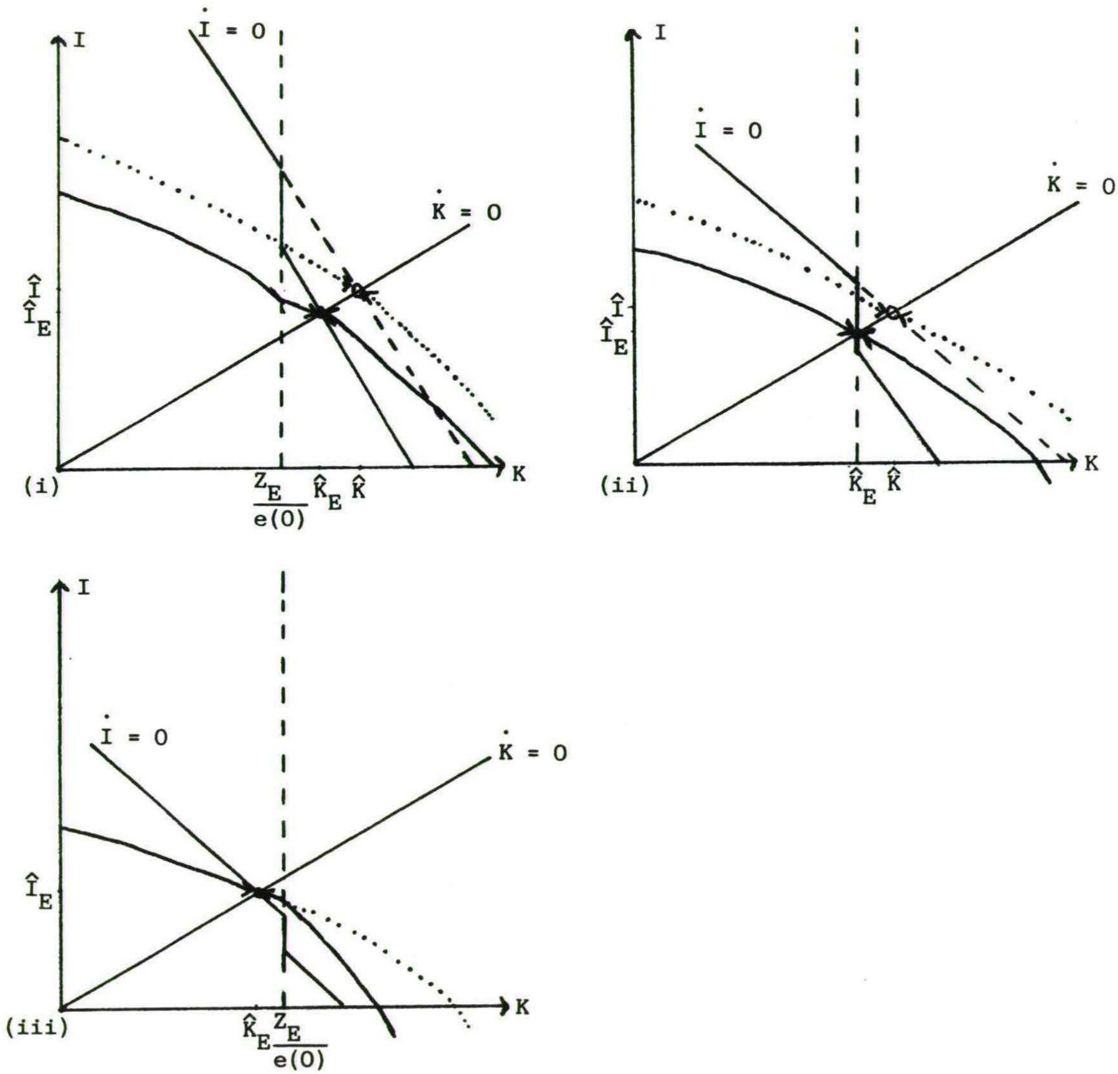


Figure 1



(i)
$\dot{I} = 0$
$\dot{K} = 0$
I
K
$\hat{I}$
$\hat{I}_E$
$\frac{Z_E}{e(0)}$
$\hat{K}_E$
$\hat{K}$
(ii)
$\dot{I} = 0$
$\dot{K} = 0$
I
K
$\hat{I}$
$\hat{I}_E$
$\hat{K}_E$
$\hat{K}$
(iii)
$\dot{I} = 0$
$\dot{K} = 0$
I
K
$\hat{I}_E$
$\hat{K}_E$
$\frac{Z_E}{e(0)}$

Figure 2

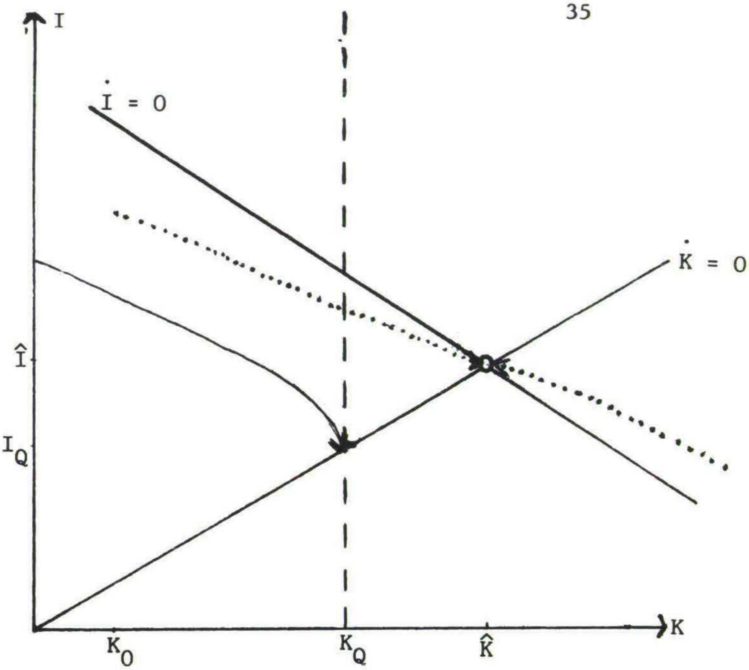


Figure 3

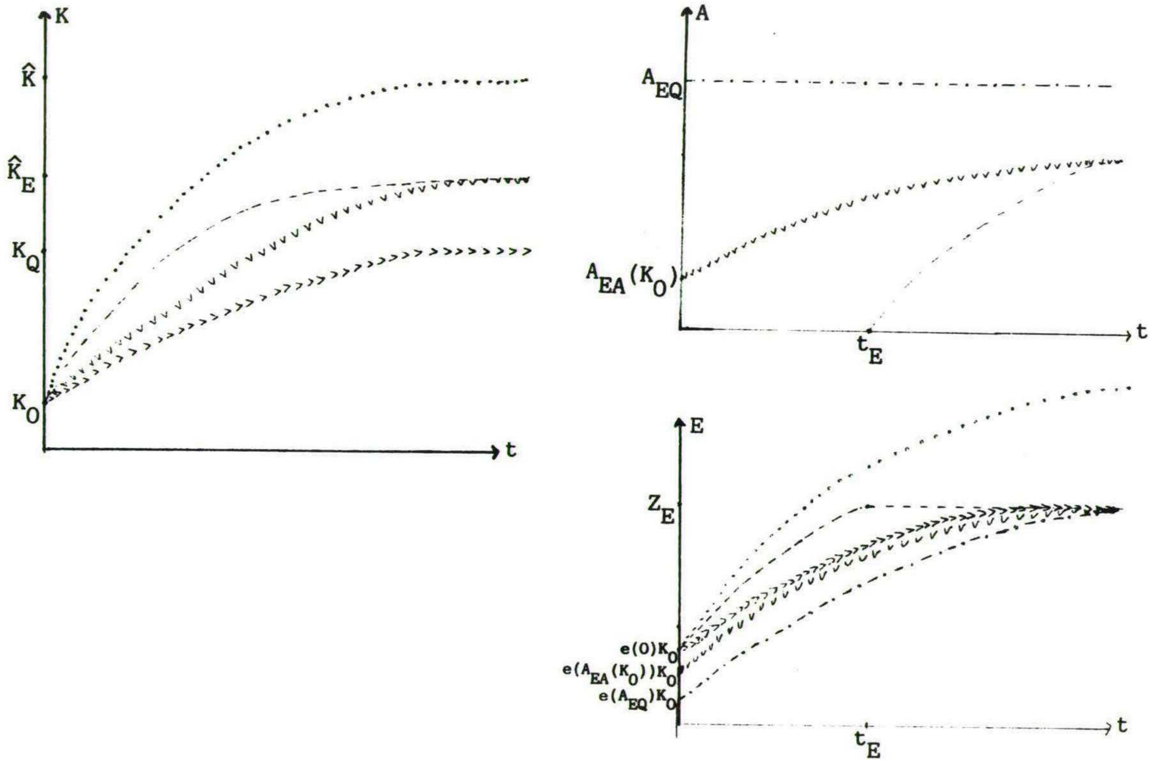

K
$\hat{K}$
$\hat{K}_E$
K_Q
K_0
t
A
A_{EQ}
$A_{EA}(K_0)$
t
t_E
E
Z_E
$e(0)K_0$
$e(A_{EA}(K_0))K_0$
$e(A_{EQ})K_0$
t
t_E

Figure 4

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